Shooting and Bouncing Rays: Calculating the RCS of an Arbitrarily Shaped Cavity

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Abstract—A novel ray-shooting approach is presented for calculating the interior radar cross section (RCS) from a partially open cavity. A dense grid of rays is launched into the cavity through the opening. The rays bounce from the cavity walls based on the laws of geometrical optics and eventually exit the cavity via the aperture. An innovative scheme of physical optics is then applied to compute the backscattered field from the exit rays. This method is so simple in concept that there is virtually no restriction on the shape or material loading of the cavity. Numerical results obtained by this method are compared with those for the modal analysis for a circular cylinder terminated by a PEC plate. RCS results for an S-bend circular cylinder are generated on the Cray X-MP and show significant RCS reduction. Finally, some of the limitations and possible extensions of this technique are discussed.

I. INTRODUCTION

THE PROBLEM of scattering from open-ended cavities is of great importance in both radar cross section (RCS) reduction and electromagnetic pulse (EMP) coupling studies. Traditionally, the problem of calculating the RCS of a cavity structure is treated by the modal analysis. For example, Chang and Senior [1] studied the open spherical shell problem based on the expansion of interior and exterior fields in the spherical wave function. Recently, the problem of scattering from an open-ended circular cylinder with wall coatings was analyzed by Lee et al. [2] by utilizing the cylindrical waveguide modes. Analytic solutions to a family of canonical problems have also been obtained by the dual series approach [3], [4]. However, the above approaches may become cumbersome if 1) the shape of the cavity is not a perfect sphere or cylinder, 2) the electrical dimension of the cavity is large, or 3) the space inside the cavity is not homogeneous. These restrictions prevent the realistic modeling of physical problems.

In this paper, a different strategy for analyzing the open cavity problem, entitled shooting and bouncing rays (SBR), is presented. A dense grid of geometric optics (GO) rays representing an incident plane wave is "shot" into the cavity through the front aperture and followed as the rays bounce from conductors, penetrate through materials, and eventually return to the opening of the cavity (see Fig. 1). An innovative scheme is then used to integrate the aperture field to obtain the scattered field. This approach has the feature that a real physical problem can be modeled closely, taking into account the noncircular opening of the cavity, the wall coating, and the longitudinal bending or twisting of the cavity. It is so simple in concept that there is virtually no restriction on the shape or material loading of the cavity.

This paper is organized as follows. In Section II, the problem is formulated based on geometrical optics. The paths of each individual ray are first determined by Snell's law. The field amplitudes associated with each ray are computed by taking into consideration 1) geometrical divergence factor, 2) polarization, and 3) material loading of the cavity walls. The contributions to the backscattered field from individual rays are then summed up to arrive at the total RCS due to the interior irradiation of the cavity. In Section III, extensive numerical results for various cavity geometries are presented. They are compared to the existing modal results for the case of a circular cylinder with a PEC termination. RCS results for an S-bend structure generated on the Cray X-MP supercomputer are also presented. Finally, some of the limitations of this approach are addressed.

II. FORMULATION OF SHOOTING AND BOUNCING RAYS

Referring to Fig. 1, consider an arbitrary cavity with an opening at the aperture \( \Sigma_t \). The inside walls of the cavity may be coated with dielectric or magnetic materials. Additional scatterers may also exist inside the cavity but will not be considered in this treatment. The incident plane wave is given by (for \( \exp (j\omega t) \) time convention)

\[
\mathbf{E} = \left[ -\hat{\mathbf{r}} \hat{\mathbf{I}} + \hat{\mathbf{I}} \hat{\mathbf{r}} \right] e^{j\mathbf{E} \cdot \mathbf{r}}
\]  

(1)

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where

\[ \begin{align*}
\vec{I} & = k_0(\hat{x} \sin \theta^i \cos \phi^i + \hat{y} \sin \theta^i \sin \phi^i + \hat{z} \cos \theta^i) \\
I & = \text{amplitude of the perpendicular (or vertical) polarization} \\
\vec{I} & = \text{amplitude of the parallel (or horizontal) polarization.}
\end{align*} \]

The problem at hand is to determine the backscattered field in the direction of \( \vec{I} \) and the RCS of the cavity. The backscattering from the exterior of the cavity is not treated, since 1) the exterior scattering is small for small angle of incidence \( \theta^i \) and 2) the exterior scattered field can be considered separately. In addition, it is well known that the contribution to the backscattered field from the energy coupled into and re radiated from the cavity is much greater than from the rim diffraction, especially if the cavity opening is large compared to the wavelength [5]–[9]. Therefore, attention will be focused on the interior irradiation mechanism.

The ray bouncing approach will be carried out in three parts.

1) Given the geometry of the cavity and the incident field, find the ray paths in the cavity by ray tracing. This part of the problem is dependent only on the geometry of the problem.

2) Determine the field amplitude of the exit rays on the aperture based on geometrical optics. This involves calculating the ray tube divergence factors and the reflection coefficients.

3) Use Kirchhoff’s approximation (physical optics) to determine the backscattered field and the RCS.

These steps will be described in detail in the following sections.

### A. Ray Tracing

In order to model the incident plane wave, parallel rays are launched from the incident direction. Each ray is represented by a line in space with a reference point \((x_0, y_0, z_0)\) and a direction vector \((s_x, s_y, s_z)\). Any point \((x_1, y_1, z_1)\) along this line would then be described by

\[ (x_1, y_1, z_1) = (x_0, y_0, z_0) + (s_x, s_y, s_z)t \quad (2) \]

where the parameter \(t\) can be conveniently thought of as time. By this definition, if the phase of the field at \((x_1, y_1, z_1)\) lags that of point \((x_0, y_0, z_0)\), then \(t\) will be a positive quantity. The direction vector of the incident rays is given by

\[ \begin{align*}
  s_x & = -\sin \theta^i \cos \phi^i \\
  s_y & = -\sin \theta^i \sin \phi^i \\
  s_z & = -\cos \theta^i.
\end{align*} \quad (3) \]

The reference point \((x_0, y_0, z_0)\) on the incident plane \(\Sigma^i\) can be related to the point \((x_0, y_0, 0)\) on the aperture \(\Sigma_0\) via

\[ \begin{align*}
  x_0 & = (s_x^2 + s_z^2)x_0 - s_ys_0y_0 + s_zt_0 \\
  y_0 & = (s_x^2 + s_z^2)y_0 - s_ys_0x_0 + s_zt_0 \\
  z_0 & = -s_ys_0x_0 - s_ys_0y_0 + s_zt_0.
\end{align*} \quad (4) \]

The parameter \(t_0\) in (4) determines how far \(\Sigma^i\) is from the aperture plane and can be chosen arbitrarily (e.g., \(t_0 = -10\)). To summarize, the incident ray is described by (2), where the direction vector is given in (3) and the reference point \((x_0, y_0, z_0)\) can be related to a corresponding point on the aperture via (4).

Once the incident rays have been defined, the impact point of each ray on the inner wall of the cavity can be determined. This is accomplished by solving the equation describing the cavity \(z = f(x, y)\) and (2). For example, if the cavity is a circular cylinder with radius \(a\), the intersection is found by substituting into (2)

\[ t = (-B + \sqrt{B^2 - 4AC})/2A \quad (5) \]

where

\[ \begin{align*}
  A & = s_x^2 + s_y^2 \\
  B & = 2(s_xx_0 + s_yy_0) \\
  C & = x_0^2 + y_0^2 - a^2.
\end{align*} \]

Next, the equation for the reflected ray will be determined. The reflected ray must satisfy Snell’s law, namely, 1) it must lie in the plane of incidence and 2) the angle of reflection must equal the angle of incidence. Referring to Fig. 2, define a unit vector

\[ \hat{m} = (\hat{1} \times \hat{n})/\sin \theta^i \quad (6) \]

where \(\hat{1}\) is the unit vector pointing from point 1 to point 0. Note that \(\hat{m}\) is perpendicular to the plane of incidence. The above two conditions can be restated as 1) \(\hat{1} \cdot \hat{m} = 0\) and 2) \(\hat{1} \cdot \hat{n} = \cos \theta^i\). These two conditions are solved simultaneously in order to determine the direction of the reflected ray. The equation for the reflected ray is now complete since both the direction and the reference point (impact point) are known.

Consequently, by using this reflected ray as the incident ray, the above procedure can be repeated until the ray exists the cavity. The exit of the ray from the cavity is easily
detected when the ray intersects the aperture $S_a$. For every ray launched, a set of impact points inside the cavity and the direction of the exiting ray are obtained.

**B. Amplitude Tracking**

Once the ray paths inside the cavity are found, the field amplitude along the ray can be determined. For each ray launched, a set of impact points was found within the cavity, $\{(x_i, y_i, z_i)\}$, $i = 1, 2, \cdots, N$. The aperture field associated with the exit ray $\vec{E}(x_N, y_N, 0)$, is to be determined.

From geometrical optics, the electric field obeys the following recursion relationship:

$$
\vec{E}(x_{i+1}^-, y_{i+1}^-, z_{i+1}^-) = (DF)_i \cdot (\bar{F})_i \cdot \vec{E}(x_i^-, y_i^-, z_i^-) \cdot e^{-j(\text{phase})}
$$

where phase = $k_0 [(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2]^{1/2}$ and $\vec{E}(x_i^-, y_i^-, z_i^-)$ is the incident field at $(x_i, y_i, z_i)$. $(DF)_i$ is the planar reflection coefficient matrix at the $i$th reflection point where the original curved interface in Fig. 2 is replaced by its tangent plane at the reflection point. $(DF)_i$ is the divergence factor which governs the spreading of the differential ray tube from just after the $i$th reflection to just before the $(i + 1)$th reflection. The remaining tasks are the determination of these two quantities.

1) **Planar Reflection Coefficients:** Referring to Fig. 3(a), attention will be restricted to the case of one layer of dielectric or magnetic coating with thickness $\gamma$ and backed by perfect conducting walls. The transmission line analogy to the field problem is shown in Fig. 3(b). The well-known planar reflection coefficients for the transverse electric (TE) and the transverse magnetic (TM) case are summarized in Table I. For the assumed time convention, the complex root is chosen to have a positive real component and a negative imaginary component.

<table>
<thead>
<tr>
<th>TABLE I REFLECTION COEFFICIENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE</td>
</tr>
<tr>
<td>$\Gamma$ = $\frac{1 - Z_0 Z_{\infty}}{1 + Z_0 Z_{\infty}}$</td>
</tr>
<tr>
<td>$Z_+ = Z_0 \tan (\beta_0 \gamma)$</td>
</tr>
<tr>
<td>$Z_1 = \mu_1 \beta_1$</td>
</tr>
<tr>
<td>$Z_0 = \omega \mu_0 / \beta_1$</td>
</tr>
</tbody>
</table>

$$
\beta_0 = \sqrt{\mu_0 \varepsilon_0 \cos \theta_i^c} \\
\beta_1 = \sqrt{\mu_1 \varepsilon_1 \mu_0 \varepsilon_0 \sin^2 \theta_i^c}
$$

2) **TE/TM Decomposition:** In order to apply the reflection coefficients $\Gamma$ and $\bar{\Gamma}$ found in Table I, the incident field needs to be decomposed into its TE and TM components. Using subscript $c$ to denote local coordinates (see Fig. 2), the incident field can be written as

$$
\vec{E}(1^-) = (\vec{E}_i^- \cdot \hat{\phi}_i^c) \hat{\phi}_c^i + (\vec{E}_i^- \cdot \hat{\phi}_c^i) \hat{\phi}_c^i.
$$

The reflected field is then given by

$$
\vec{E}(1^+) = \Gamma (\vec{E}_i^- \cdot \hat{\phi}_c^i) \hat{\phi}_c^i + \bar{\Gamma} (\vec{E}_i^- \cdot \hat{\phi}_c^i) \hat{\phi}_c^i.
$$

So the reflected field is easily found once $\hat{\phi}_c^i$ and $\hat{\phi}_c^i$ are determined in terms of the global $xyz$ coordinate. Choose

$$
\begin{align*}
\hat{x}_c &= \hat{m} \times \hat{n} \\
\hat{z}_c &= -\hat{n} \\
\hat{y}_c &= -\hat{m}.
\end{align*}
$$

From Fig. 2,

$$
\begin{align*}
\theta_i^c &= \cos^{-1} (\hat{m} \cdot \hat{n}) \\
\phi_i^c &= 0.
\end{align*}
$$

Then

$$
\begin{align*}
\hat{\phi}_c^i &= \hat{x}_c \cos \theta_i^c - \hat{z}_c \sin \theta_i^c \\
\hat{\phi}_c^i &= \hat{y}_c
\end{align*}
$$

and

$$
\begin{align*}
\hat{\phi}_c^i &= -\hat{x}_c \cos \theta_i^c - \hat{z}_c \sin \theta_i^c \\
\hat{\phi}_c^i &= \hat{y}_c.
\end{align*}
$$

3) **Divergence Factor:** The detailed derivation of the divergence factor for a curved interface can be found in [10]–[12]. Only the pertinent results will be included here. First, three sets of basis vectors are chosen:

incident wavefront

$$
\begin{align*}
\vec{x}_i^1 &= \hat{n} \\
\vec{x}_i^2 &= \hat{x}_i^1 \times \hat{n}
\end{align*}
$$
\[
\begin{align*}
\text{reflected wavefront:} & \quad \left\{ \begin{array}{l}
\hat{\xi}'_1 = \hat{m} \\
\hat{\xi}'_2 = \hat{z}' \times \hat{\xi}'_1 \\
\end{array} \right. \\
& \quad (15) \\
\text{curved surface:} & \quad \left\{ \begin{array}{l}
\hat{N}' = \hat{n} \\
\hat{\xi}_1 = \hat{r}_u \\
\hat{\xi}_2 = \hat{n} \times \hat{\xi}_1. \\
\end{array} \right. \\
& \quad (16)
\end{align*}
\]

The curved surface is described by \( \mathbf{r}(u, v) = (x(u, v), y(u, v), z(u, v)) \). \( \hat{r}_u \) denotes the first derivative of \( \mathbf{r} \) with respect to \( u \). The curvature matrices \( \hat{Q}'_1, \hat{Q}'_2, \) and \( \hat{Q}' \) associated with the above basis vectors will now be determined. The curvature matrix for the incident plane wave evaluated at a point 1 is simply given by

\[
\hat{Q}'(i \pm 1) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \\
\]

The curvature matrix of the surface is given by

\[
\hat{Q}'(i) = \begin{bmatrix} \hat{Q}'_{11} & \hat{Q}'_{12} \\ \hat{Q}'_{21} & \hat{Q}'_{22} \end{bmatrix} \\
\]

where

\[
\hat{V} = \begin{bmatrix} \hat{r}_u \cdot \hat{\xi}_1 & \hat{r}_u \cdot \hat{\xi}_2 \\ \hat{r}_v \cdot \hat{\xi}_1 & \hat{r}_v \cdot \hat{\xi}_2 \end{bmatrix} \\
\hat{Q}'_{11} = \frac{eG - fF}{E - F^2}, \quad \hat{Q}'_{12} = \frac{fE - eF}{E - F^2} \\
\hat{Q}'_{21} = \frac{fG - gF}{E - F^2}, \quad \hat{Q}'_{22} = \frac{gE - fF}{E - F^2} \\
E = \hat{r}_u \cdot \hat{r}_u, \quad F = \hat{r}_u \cdot \hat{r}_v, \quad G = \hat{r}_v \cdot \hat{r}_v \\
e = \frac{\hat{r}_{uv} \cdot (\hat{r}_u \times \hat{r}_v)}{\sqrt{E - F^2}}, \quad f = \frac{\hat{r}_{uv} \cdot (\hat{r}_v \times \hat{r}_u)}{\sqrt{E - F^2}} \\
g = \frac{\hat{r}_{uv} \cdot (\hat{r}_u \times \hat{r}_v)}{\sqrt{E - F^2}}.
\]

The curvature matrix for the reflected wavefront is found by solving for \( \hat{Q}' \) in the following equation enforcing the phase matching condition of the incident and reflected wavefronts at the reflection surface:

\[
(\hat{\bar{P}}')^T \hat{Q}' \hat{\bar{P}}' + 2p_{33}' \hat{Q}'^2 = (\hat{\bar{P}}')^T \hat{\bar{Q}}' \hat{\bar{P}}'. \\
\]

where

\[
\hat{\bar{P}}' = \begin{bmatrix} \hat{\xi}'_1 & \hat{\xi}'_2 \\ \hat{\xi}'_2 & \hat{\xi}'_1 \end{bmatrix} \\
\]

and

\[
p_{33}' = \hat{z}' \times \hat{n}.
\]

Let \( Q_{mn}' \) be the \((m, n)\)th element of \( \hat{Q}' \). The two principal radii of curvature \((R_{1}'', R_{2}'')\) of the reflected wavefront are given by

\[
\frac{1}{R_{1}''} = \frac{1}{2} \left\{ \begin{array}{l}
(Q_{11}' + Q_{22}') \\
\pm \sqrt{(Q_{11}' + Q_{22}')^2 - 4(Q_{11}'Q_{22}' - Q_{12}'Q_{21}')}. \end{array} \right. \\
(20)
\]

Finally, the divergence factor from the reflection point 1 to the observation point 2 is computed from

\[
(DF)_1 = \frac{1}{\sqrt{1 + (12/R_1')}} \frac{1}{\sqrt{1 + (12/R_2')}}. \\
(21)
\]

The square roots in (21) are to be taken as either positive real or negative imaginary. Based on this sign convention, the phase of the divergence factor can be 0, \(\pi/2\), or \(\pi\), depending on whether there are zero, one, or two foci between points 1 and 2. This particular choice of square root is consistent with the phase anomaly associated with the focal region fields [13]. It is worthwhile to point out here that in shaped reflector problems, attempts have been made at computing the magnitude of the divergence factor by tracing differential ray tubes [14]. In the present open cavity problem, the highly irregular geometry will cause many ray crossings (see Fig. 4) and the above-mentioned technique will not predict the correct phase for the divergence factor.

Once the incident field at point 2 is found, let

\[
\hat{Q}'(i = 2) = \{[\hat{Q}'(i = 1)]^{-1} + 12\hat{I}\}^{-1}. \\
(22)
\]

\( \hat{I} \) in the above expression is the \(2 \times 2\) identity matrix. \( \hat{Q}'(i = 2) \) is the curvature matrix of the incident wavefront evaluated at point 2 and is defined with respect to the basis vectors described in (15). Finally, the above procedure for calculating the divergence factor can be repeated for all subsequent reflections inside the cavity until the geometrical optics field on the aperture is found.

**C. Physical Optics**

Given the outgoing field on the aperture \( \Sigma_A \), the backscattered field can be computed by the standard physical-optics approximation. First, the outgoing field is replaced by an equivalent magnetic current sheet \( \hat{K}' \):

\[
\hat{K}' = \begin{cases} 
2\hat{E}(\hat{x}_N, \hat{y}_N, 0) \times \hat{z}; & \text{over } \Sigma_A \\
0; & \text{outside } \Sigma_A.
\end{cases} \\
(23)
\]

\( \hat{K}' \) radiates in the backscattering direction and gives rise to
TABLE II
RADAR CROSS-SECTION DEFINITION

<table>
<thead>
<tr>
<th>1-polarization ((l = 1, \tilde{l} = 0))</th>
<th>1-polarization ((l = 0, \tilde{l} = 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-pol</td>
<td>(RCS_{90} = 4\pi</td>
</tr>
<tr>
<td>Cross-pol</td>
<td>(RCS_{90} = 4\pi</td>
</tr>
<tr>
<td>Phase</td>
<td>(\angle A_q \leq \angle A_0)</td>
</tr>
</tbody>
</table>

the RCS. The backscattered field is calculated from

\[
\vec{E}_{bs} = \frac{e^{-jk_0r}}{r} \begin{bmatrix} [\tilde{A}_0] \\
[\tilde{A}_\phi] \end{bmatrix}
\]

\[
[\tilde{A}_0] = \frac{jk_0}{2\pi} \sum_{\text{exit ray}} \iint_{\Sigma_A} \text{d}x\text{d}y \cdot e^{jk_0(xu + yv)} \left[ E_x \cos \phi^i + E_y \sin \phi^i \right] \left[ (-E_x \sin \phi^i + E_y \cos \phi^i) \cos \theta^i \right]
\]

\[u = \sin \theta^i \cos \phi^i\]
\[v = \sin \theta^i \sin \phi^i\]

(24)

\(E_x\) and \(E_y\) are the x and y components of the outgoing field on the aperture \(\Sigma_A\). The RCS is defined in Table II.

In general, the positions of the outgoing rays are nonuniformly dispersed over the aperture, even if the incident rays are launched uniformly. Since the outgoing rays do not lie on an equally spaced grid, the integration in (24) cannot be easily carried out. One possible solution to this problem would be to interpolate the results to find the fields on the integration grid. However, the proper interpolation scheme is difficult and an alternative approach is taken. Suppose only a small ray tube is shot into the cavity. This ray tube bounces around the walls and eventually comes to the aperture plane. It is possible to compute the backscattered field due to this ray tube by taking into account its wavefront curvature, size, and shape. By repeating this process until enough ray tubes are launched into the cavity to model the incident plane wave, the total scattered field should be the sum of the scattered field due to each individual ray tube. Using this idea, the scattered field expression will now be derived.

Consider one of the incident ray tubes with an area of \((\Delta x, \Delta y)\). The central ray with direction vector \((s_x, s_y, s_z)\) hits point \((x_0, y_0)\) on \(\Sigma_A\) (Fig. 4). The field within the exit ray tube area \((\Delta x_1, \Delta y_1)\) will be approximated as follows:

\[
\begin{bmatrix} E_x(x_1, y) \\
E_y(x_1, y) \end{bmatrix} = \begin{bmatrix} E_x(x_1, y_1) \\
E_y(x_1, y_1) \end{bmatrix} e^{-jk_0r(x-x_1) + s_z(y-y_1)}.
\]

(25)

In other words, the field within the ray tube at point \((x, y)\) has the same magnitude as the field associated with the central ray. In addition, there is a linear phase variation across the ray tube. This linear phase approximation should be valid as long as the output ray tube is not too large. The physical-optics integral in (24) can now be evaluated using the plane-wave approximation. By summing over each ray tube, (24) becomes

\[
[\tilde{A}_0] = \frac{jk_0}{2\pi} \sum_{\text{exit ray}} \iint_{\Sigma_A} \text{d}x\text{d}y \cdot e^{jk_0(xu + yv)} e^{-jk_0r(x-x_1) + s_z(y-y_1)} [E_x(x_1, y_1) \cos \phi^i + E_y(x_1, y_1) \sin \phi^i]
\]

\[-E_x(x_1, y_1) \sin \phi^i \cos \theta^i + E_y(x_1, y_1) \cos \phi^i \cos \theta^i\]

(26)

Since the fields associated with each exit ray, \(E_x(x_1, y)\) and \(E_y(x_1, y)\), are independent of the integration variables \(x\) and \(y\), the bracket in (26) can be taken out of the integral sign:

\[
[\tilde{A}_0] = \frac{jk_0}{2\pi} \sum_{\text{exit ray}} \left[ E_x(x_1, y_1) \cos \phi^i + E_y(x_1, y_1) \sin \phi^i \right]
\]

\[-E_x(x_1, y_1) \sin \phi^i \cos \theta^i + E_y(x_1, y_1) \cos \phi^i \cos \theta^i\]

\[e^{+jk_0r(x-x_1) + s_z(y-y_1)} (\Delta x_1, \Delta y_1) I_i\]

(27)

where

\[I_i = \frac{1}{(\Delta x_1, \Delta y_1)} \iint_{\Sigma_A} \text{d}x\text{d}y e^{jk_0r(x-x_1) + s_z(y-y_1)}\]

and

\[(\Delta x_1, \Delta y_1) = \text{area of the exit ray tube.}\]

Closer examination reveals that \(I_i\) is nothing but the Fourier transform of the ray tube shape function (normalized with respect to the ray tube area). In order to determine the shape of the \(i\)th exit ray tube, four adjacent rays are launched into the cavity in the form of a square as shown in Fig. 5. These four rays, upon exit, form a tetragonal shape on the output aperture.
The position vectors of the four rays on the aperture are denoted by \( \mathbf{r}_n = x_n \mathbf{x} + y_n \mathbf{y}, \) \( n = 1, 2, 3, 4. \) The normalized Fourier transform of the tetragonal shaped function can now be evaluated as described in [15]:

\[
I_i = S(p, q)/S(0, 0)
\]  

(28)

where

\[
S(p, q) = \sum_{n=1}^{4} e^{j(p x_n + q y_n)} \frac{(x_{n+1} - x_n)(y_n - y_{n-1}) - (y_{n+1} - y_n)(x_n - x_{n-1})}{[(x_n - x_{n-1})p + (y_n - y_{n-1})q]}
\]

\[
S(0, 0) = \frac{(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_4 - x_4 y_3) + (x_4 y_1 - x_1 y_4)}{2}
\]

\[p = k_0(u - s_x)\]

\[q = k_0(v - s_y)\]

\[\bar{w} = px + qy\]

Note that by definition, \( \mathbf{r}_0 = \mathbf{r}_4 \) in the above expression. The shape of the ray tube is approximated by a four-sided polygon. If desired, higher order polygons can always be used for a more refined approximation of the ray tube shape.

Lastly, the output ray tube area \( (\Delta x_i \Delta y_i) \) can be related to the incident ray tube area \( (\Delta x_0 \Delta y_0) \) through the ray tube divergence factors calculated earlier. The area of the exit ray tube is related to the area of the incident ray tube via the product of the divergence factors:

\[(\Delta x_i \Delta y_i) = \left[ \prod_{i=1}^{N} (DF_i) \right]^{-2} \left( \frac{\cos \theta_1}{\cos \theta_2} \right) (\Delta x_0 \Delta y_0).
\]  

(29)

The ratio of the cosines is an added factor which accounts for the angles of the ray tubes entering and exiting the cavity (see Fig. 6).

If the quantity of interest is the bistatic cross section (BCS)
rather than the assumed monostatic RCS, the incident direction angles $\theta'$ and $\phi'$ should be replaced by the observation direction angles $\theta^0$ and $\phi^0$ in (24), (26), and (27).

To summarize, the physical-optics integral defined in (24) is evaluated by summing the backscattering contributions due to each ray tube. The wavefront of the exiting ray tube is approximated to be planar as given in (25). The shape of the exit ray tube is approximated by a four-sided polygon. The far field is proportional to the Fourier transform of this normalized shape function and is found using (28). Finally, the area of the exit ray tube is found by the product of the divergence factors and is related to the incident ray tube via (29).

III. Numerical Results

A ray bouncing program, denoted as SBR, has been implemented based on the formulation described in Section II. In this program, it is assumed that the cavity is composed of an arbitrary cylindrical cavity with a flat aperture at $z = 0$ and a flat PEC termination at $z = d$. The inner walls of the cylinder can be coated with either dielectric or magnetic materials. Below, the comparison of SBR results with those generated by the modal analysis for a circular cylinder with uniform cross section is shown. Representative results for an S-bend structure are also presented.

A. Exit Ray Positions

Shown in Fig. 7 are plots of the incident and exit ray positions on the aperture plane for a circular cylinder with a flat end-plate. The incident rays are launched from a uniformly spaced grid on the aperture. For a small incident angle ($\theta = 1^\circ$), the majority of the incident rays simply reflect off the end-plate and bounce back to the aperture without hitting the side walls. Therefore, they remain uniformly spaced on the exit aperture. As the incident angle increases, the positions of the exit rays become more dispersed due to the large number of bounces inside the cavity. This behavior is even more apparent in the case of an elliptical cylinder with nonuniform cross sections as shown in Fig. 8.

B. Comparison with Modal Analysis

RCS results generated by the SBR program are compared with those obtained by the modal analysis [2] for a straight circular cylinder with PEC walls. Shown in Figs. 9-11 are RCS plots versus angle $\theta$ for circular cylinders with a fixed diameter of $4 \lambda$ and a different depth $d$. The copolarization of both the perpendicular (vertical) and parallel (horizontal) cases are plotted as $\text{RCS}_{\theta 0}$ and $\text{RCS}_{\phi 0}$, respectively. The solid circles are the modal results and the solid curves represent results from the present SBR formulation. Good agreement is obtained between the ray bouncing method and the modal analysis, except in Fig. 11(a) for the $10 \lambda$ deep cylinder. Possible reasons for the discrepancy will be addressed in Section III-D.

The convergence of the present ray formulation is checked by increasing the total number of rays launched into the cavity. Shown in Fig. 12(a) is the $\text{RCS}_{\theta 0}$ at $\theta = 25^\circ$ plotted as a function of the number of rays per wavelength. It is observed that the results converged nicely for a density greater than about ten rays per wavelength. As the depth of the cylinder increases, the rate of convergence slows down. This is presumably due to an increased number of ray bounces inside the cavity and the highly divergent exit ray directions on the out-
put aperture. In Fig. 12(b), the RCS at $\theta = 50^\circ$ is plotted as a function of the number of rays. At this larger angle, convergence is slower compared to the case in Fig. 12(a), again due to the increased number of bounces.

C. Radar Cross Section of Large Circular Cylinders

Shown in Figs. 13 and 14 are the RCS plots of large circular cylinders. In Fig. 13, the cylinder has a diameter of $10\lambda$ and depth of $10\lambda$. In Fig. 14, the cylinder has a diameter of $20\lambda$ and depth of $40\lambda$. Even better agreement between the ray-bouncing results and the modal results is observed as compared to those in Figs. 9–11. This confirms our intuition that the higher the frequency, the better the ray optics approximation. The ray results were generated on the Cray X-MP supercomputer available to us at both the University of Texas and the University of Illinois. The data in Fig. 14 took approximately 20 seconds of CPU time per point based on a ten rays per wavelength criterion found in Section III-B. The computer code did not take advantage of the vector processing capability of the X-MP.

Next, an S-bend structure shown in Fig. 15 is studied. The S-bend has a uniform circular cross section with an offset of exactly one diameter between the aperture and the end-plate. The center curve of this structure is described by a half-cosine function:

$$y = a [\cos (\pi x / d') - 1].$$

The RCS results are shown in Fig. 16 for 1) a straight cylinder ($a = 5\lambda$ and $d' = 20\lambda$), 2) an S-bend with PEC interior,
Fig. 11. Comparison of ray bouncing with modal analysis for a circular cylinder with diameter $4\lambda$ and depth $10\lambda$. (a) $\phi$-polarization. (b) $\theta$-polarization.

Fig. 12. Convergence of the RCS results as the ray density launch is increased. (a) $\theta = 25^\circ$. (b) $\theta = 50^\circ$.

and 3) an S-bend with a single layer of lossy dielectric coating ($\varepsilon_r = 1.5 - j2.0$, thickness $\tau = 0.25\lambda$). Significant RCS reduction is observed in case 3).

D. Limitations of Ray Bouncing

The present ray bouncing method is based on a high-frequency picture, namely, the incident plane wave retains its ray optical behavior once inside the cavity. The underlying idea is that it is much easier to keep track of the rays than to determine the hopelessly complicated cavity modes. In order to investigate the limitations of this technique, the problem of a plane wave impinging on an open-ended parallel-pate waveguide was studied [16]. Shown in Figs. 17(a) and 17(b) are plots of the energy flow inside the guide for a waveguide with separation $75\lambda$ and a plane wave incident at $30^\circ$. The physically intuitive picture based on the ray argument is shown in Fig. 17(a) and depicts a beamlike propagation down the guide. The actual energy flow computed by summing the waveguide modes is shown in Fig. 17(b). The resemblance of the actual distribution to the ray picture is apparent in this case. Therefore, the ray bouncing method should indeed give a very accurate description of the actual problem. As the waveguide separation is decreased to $5\lambda$, however, we observe that the actual field begins to blur after propagating in a beamlike manner some distance into the guide as shown in Fig. 18. The simple ray-optical picture of fields breaks down at this point. Therefore, the present technique should be valid as long as the opening is large compared to the wavelength, as expected. In order to modify the simple ray picture to account for the beam
blurring, the incorporation of edge diffraction effects may be necessary [17], [18].

IV. SUMMARY

A ray bouncing method was presented for calculating the electromagnetic scattering from an arbitrary open cavity. This approach is based on tracking a large number of rays launched into the cavity through the opening and determining the geometrical optics field associated with each ray by taking into consideration 1) the geometrical divergence factor, 2) polarization, and 3) material loading of the cavity walls. The contributions to the backscattered field from individual rays launched into the cavity are summed up to obtain the total internal RCS of the cavity. It was shown that the RCS results obtained by the ray approach agree well with those for the modal analysis even for a small 4 λ cavity. Results for a 20 λ circular cylinder as well as an S-bend structure were presented. Finally, the limitations of the present approach are
addressed through a simple parallel-plate waveguide problem. It was found that for an aperture opening large compared to the wavelength, the ray optical description on which the ray bouncing technique is based is completely adequate. As the opening is decreased, additional features of the actual field must be accounted for. This topic is currently under consider-

Fig. 16. RCS of the uncoated and coated S-bends. (a) φ-polarization. (b) θ-polarization.

Fig. 17. Plots of Poynting vectors inside an open-ended parallel-plate waveguide (25 λ) with plane wave excitation based on (a) ray-optical description of the field, (b) the actual fields based on modal expansion.

References

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