Reliability-Based Design Optimization of Stiffened Panels

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Abstract
This paper explores reliability-based designs of isogrid stiffened panels. Uncertainties in material properties and geometric manufacturing uncertainties are represented by random variables. Due to the multiple failure modes in the stiffened panels, polynomial response surface approximation are fit to the most critical safety margins in the panel. Probability of failure is calculated by Monte Carlo simulation using the polynomial response surfaces. A probabilistic sufficiency factor approach is employed to facilitate the design optimization.

1. Introduction
Stiffened panels are often used in aircraft and launch vehicle design to obtain lightweight structures with high bending stiffness. The design optimization of stiffened panels under buckling and strength constraints is characterized by a large number of local optima (Lamberti, et al. 2003). Some of these designs are more sensitive to uncertainties than the others. Therefore, it is important to provide designer a reliability-based optimum panel design.

This paper presents reliability-based designs of isogrid panels. The problem is to minimize the weight of the stiffened panel subject to a reliability constraint. The reliability-constraint is evaluated by Monte Carlo simulation, requiring a large number panel analyses. Due to the limitation of computer resources and time, PANDA2 software (Bushnell, [1]) is employed to analyze stiffened panels. PANDA2 uses a combination of approximate physical models, exact closed form (finite strip analysis) models and 1-D discrete branched shell analysis models to calculate pre-buckling, buckling and post-buckling responses. PANDA2 also provides deterministic global optimization based on multiple starting points strategy due to availability of fast simple analyses. The load carrying capacity of stiffened panels is greatly affected by geometric imperfections due to fabrication. The effects of geometric imperfections are taken into account in PANDA2 software directly by modifying the effective radius of cylindrical panel and indirectly by redistributing the pre-buckling stress resultants over the various segments of the panel. Various geometric imperfections, such as global, local, interring, and general ovalization of cylindrical panels can be handled by assuming the shape of buckling modal imperfection to be double trigonometric function (Bushnell, [2]).

Even with the low computational cost of PANDA2 analyses, they can not be used directly in the Monte Carlo simulation. Instead, response surface approximations are employed (Qu et al., [3] and [4]). Reliability analyses and design optimization using Monte Carlo simulation combined with response surface approximation are introduced in the next section. An example problem of the reliability-based design of isogrid stiffened panels is presented.

2. Response surface approximation for reliability-based optimization
Reliability analysis of systems with multiple failure modes often employs Monte Carlo simulation, combined with approximation to the failure evaluations.

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2.1 Monte Carlo simulation

Let \( g(x) \) denote the limit state function of a performance criterion (such as strength allowable larger than stress), so that the failure event is defined as \( g(x) < 0 \), where \( x \) is a random variable vector. The probability of failure of a system can be calculated as

\[
P_f = \int_{g(x)<0} f_X(x) \, dx
\]

where \( f_X(x) \) is the joint probability distribution function. This integral is hard to evaluate, because the integration domain defined by \( g(x) < 0 \) is usually unknown, and integration in high dimension is difficult. Commonly used probabilistic analysis methods are either moment-based methods such as the first-order-reliability-method (FORM) and the second-order-reliability-method (SORM), or simulation techniques such as Monte Carlo simulation (e.g., Melchers, [5]). Monte Carlo simulation is a good method to use for system reliability analysis with multiple failure modes.

2.2 Analysis response surface approximation to the most critical safety margins

Response surface approximations usually fit low order polynomials to the structural response in terms of random variables, \( x \),

\[
\hat{g}(x) = Z(x)^T b
\]

where \( \hat{g}(x) \) denotes the approximation to the limit state function \( g(x) \), \( Z(x) \) is the basis function vector that usually consists of monomials, and \( b \) is the coefficient vector estimated by least square regression. The probability of failure can then be calculated inexpensively by Monte Carlo simulation using the fitted polynomials.

Response surface approximations can be used in different ways. One approach is to construct local response surfaces around the most probable point that contributes most to the probability of failure of the structure. The statistical design of experiment (DOE) for this approach is iteratively performed to approach the most probable point. This local RS approach can produce satisfactory results given enough iterations. Another approach is to construct global response surfaces over the entire range of random variables, i.e., design of experiment around the mean values of the random variables. However, the reliability analysis and hence the response surface approximation needs to be performed at every design point visited by the optimizer, which requires a fairly large number of response surface constructions and thus limit state evaluations. The local response surface approximation approach is even more computationally expensive than the global approach in the design environment. Qu et al. ([3] and [4]) developed a global analysis response surface (ARS) approach in unified space of design and random variables to reduce the number of response surface approximation substantially and achieve higher efficiency than the previous approach. This analysis response surface can be written as

\[
\hat{g}(x, d) = Z(x, d)^T b
\]

where \( x \) and \( d \) are the random variable and design variable vectors, respectively. The number of response surface approximation constructed in optimization process is reduced substantially by introducing design variables into the response surface approximation formulation.

2.3 Design response surface approximation

For the reliability-based design optimization, analysis response surface (ARS) is fitted to the most critical margin in the isogrid panel in terms of both random design variables and the material random variables. Using the ARS, the probability of failure at every design point can be calculated inexpensively by Monte Carlo simulation based on the fitted polynomials. A design response surface approximation (DRS) is then fit to the probability of failure in order to filter noise generated by MCS. The details of the ARS/DRS approach are given in Qu et al. ([3] and [4]). Due to the high nonlinearity of probability of failure and safety index in the design space, a probabilistic sufficiency factor approach (Qu and Haftka, [6]) is used instead of the probability of failure in the design optimization.

3. Isogrid panel design example

An isogrid panel design problem is taken from Lamberti et al. [7] to demonstrate the reliability-based design methodology.

3.1 Reliability-based design problem formulation

Isogrid stiffened panel are cylindrical shells that have rectangular blade stiffeners positioned along the circumferential and \( \pm 60^\circ \) directions to
the circumferential directions as shown in Figure 1. The tank barrel to be optimized is stiffened externally with J-shaped ring stiffeners, and internally with a blade-shaped isogrid oriented circumferentially. The length, \( L \), of the tank barrel is 300 in, and the radius, \( r \), is 160 in. A half cylindrical tank is considered due to symmetry. The design variables are the isogrid spacing, \( b \), the isogrid blade height, \( h \), and the thicknesses the skin, \( t_1 \), and the thickness of the isogrid blades, \( t_2 \). The geometry of the ring is fixed.

Reliability-based design minimizes the weight of panel and subjected to the constraint that the probability of failure of the panel must be lower than certain give value. The reliability-based optimization of stiffened panels is formulated as

\[
\text{minimize } \quad W = W(b, h, t_1, t_2) \\
\text{such that } \quad P \leq P^u
\]

where \( b, h, t_1 \) and \( t_2 \) are the design variables; \( W \) is the weight of the panel. The reliability constraint is expressed as a limit \( P^u \) (i.e., \( P^u = 10^{-3} \)) on the probability of failure, \( P \). The four design variables are \( b, h, t_1 \) and \( t_2 \). Reliability-based optimization seeks the lightest structure satisfying the reliability constraint.

3.2 Uncertainties

The material used in the study is Al 2219-T87, which has a density of 0.1 lb/in\(^3\). The uncertainties in the material properties are represented by three random variables, two for elastic properties (\( E \) and \( \mu \)) and one for strength allowable (\( \sigma_0 \)), which are assumed to be normally distributed and uncorrelated. The mean values and coefficients of variation of the uncertainties in material properties are shown in Table 1. There are four design variables (\( b, h, t_1 \) and \( t_2 \)) which also have randomness in them due to manufacturing uncertainties, which are assumed to be uniformly distributed. Table 2 shows the percentage variation of the random design variables. These data are based on limited test data, and are intended only for illustration.

3.3 Load cases

Two load cases were used:

1. Internal proof pressure of 35 psi - critical for strength
2. Axial compressive load \( N_x = 1000 \) lb/in, with an internal (stabilizing) pressure of 5 psi – critical for buckling

3.4 Analysis response surface

The deterministic optimum obtained by global optimization in Lamberti et al. [7] is shown in Table 3. Two analysis response surfaces (ARS) were fitted to the critical margins of the two load cases in terms of seven variables, which included four design random variables, and three random variables. Statistical design of experiment is Latin Hypercube sampling (LHS, e.g., Wyss and Jorgensen, [8]), where design random variables were treated as uniformly distributed variables over the range shown in Table 4. Ranges for normal random variables are automatically handled by Latin Hypercube sampling. Using the ARS, probabilities of failure are calculated by Monte Carlo simulations.

The accuracy of the ARS is evaluated by statistical measures provided by the JMP software (Anon., [9]), which include the adjusted coefficient of multiple determination (\( R_{\text{adj}}^2 \)), and the root mean square error (RMSE) predictor. To improve the accuracy of response surface approximation, polynomial coefficients that were not well characterized were eliminated from the response surface model by using a mixed stepwise regression (e.g., Myers and Montgomery, [10]).

A quadratic polynomial of seven has 36 coefficients. The number of sampling points generated by LHS was selected to be twice the number of coefficients. Table 5 shows that the quadratic response surfaces constructed from LHS with 72 points offer good accuracy.

The deterministic design is then evaluated under material and manufacturing uncertainties. The probability of failure of the deterministic optimum under uncertainties in material properties is shown in Table 6. The dominant failure mode is local skin triangular buckling.

3.5 Design response surfaces

In order to filter the noise in the results of MCS, design response surface (DRS) are constructed to approximate the reliability constraints. Using the ARS constructed in previous section, the probability of failure at each design point of the design of experiment of DRS can be evaluated inexpensively by MCS. Since the probability of failure is highly nonlinear, the reliability constraint of Eq. (1) is
replaced by an equivalent form of constraint in terms of probabilistic sufficiency factor \( (\text{PSF, Qu and Haftka, [6]} \) as \( \text{PSF}>1 \).

The range of the cubic DRS is shown in Table 7. The error statistics are summarized in Table 8. It is seen that the accuracy of the DRS to probability of failure is poor, while DRS to PSF is accurate enough for RBDO.

3.6 Optimum panel design

Using the DRS to PSF in Table 8, reliability-based design optimization is performed. The optimum design is shown in Table 9. Its reliability evaluated by MCS using ARS with \( 10^6 \) samples is shown in Table 10. It is seen that the reliability-based design is lighter and safer than the deterministic design. The probability of failure of reliability-based design is slightly higher than the target reliability, which is due to the errors in the ARS approximation. The design can be refined by another design iteration that use a smaller design domain centered around it.

4. Concluding remarks

Reliability-based design optimization of stiffened panels is investigated. The uncertainties in the panels including both material properties and manufacturing process variation are modeled by random variables. The reliability-based design optimization is carried out using Monte Carlo simulation and response surface approximation. Due to the high nonlinearity of probability of failure, probabilistic sufficiency factor approach is employed to construct design response surface approximation. The reliability-based panel design is lighter and safer than the deterministic optimum obtained by global optimization.

Acknowledgements

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Reference


Figure 1. Isogrid-stiffened cylindrical shell with internal isogrid and external rings with isogrid pattern oriented along circumferential direction for increased bending stiffness in hoop direction

Table 1. Uncertainties in material properties (Al 2219-T87) modeled as normal random variables

<table>
<thead>
<tr>
<th></th>
<th>Young’s Modulus (E)</th>
<th>Poisson Ratio (µ)</th>
<th>Stress Allowable (σₐ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean value</td>
<td>0.107×10⁸ psi</td>
<td>0.34</td>
<td>0.58×10⁵ psi</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 2. Uncertainties in manufacturing process modeled as uniformly distributed random design variables around design value

<table>
<thead>
<tr>
<th>Percentage variation</th>
<th>b (in)</th>
<th>h (in)</th>
<th>t₁ (in)</th>
<th>t₂ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>±1%</td>
<td></td>
<td>±2%</td>
<td>±4%</td>
<td>±4%</td>
</tr>
</tbody>
</table>

Table 3. Deterministic Optimum (Lamberti et al. [7])

<table>
<thead>
<tr>
<th>b (in)</th>
<th>h (in)</th>
<th>t₁ (in)</th>
<th>t₂ (in)</th>
<th>Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.71</td>
<td>2.049</td>
<td>0.0986</td>
<td>0.1261</td>
<td>2598</td>
</tr>
</tbody>
</table>

Table 4. Range of analysis response surface approximations (inch)

<table>
<thead>
<tr>
<th>b</th>
<th>h</th>
<th>t₁</th>
<th>t₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5-10.8</td>
<td>1.9465-2.1515</td>
<td>0.09367-0.1035</td>
<td>0.1198-0.1324</td>
</tr>
</tbody>
</table>
Table 5. Quadratic analysis response surface approximation to the worst margins using Latin Hypercube sampling of 72 points

<table>
<thead>
<tr>
<th></th>
<th>Critical margins of load case 1</th>
<th>Critical margins of load case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rsquare adj.</td>
<td>0.9413</td>
<td>0.9986</td>
</tr>
<tr>
<td>RMSE predictor</td>
<td>0.0203</td>
<td>0.00273</td>
</tr>
<tr>
<td>Mean of response</td>
<td>0.4770</td>
<td>0.1986</td>
</tr>
</tbody>
</table>

Table 6. Probabilities of failure calculated by Monte Carlo simulation of $1 \times 10^6$ samples

<table>
<thead>
<tr>
<th>Probability of failure of load case 1</th>
<th>Probability of failure of load case 2</th>
<th>System probability of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>693×10^6</td>
<td>693×10^6</td>
</tr>
</tbody>
</table>

Table 7. Range of design response surface approximations (inch)

<table>
<thead>
<tr>
<th>b</th>
<th>h</th>
<th>t₁</th>
<th>t₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.6-10.7</td>
<td>1.9854 - 2.1084</td>
<td>0.0974-0.0986</td>
<td>0.1246-0.1271</td>
</tr>
</tbody>
</table>

Table 8. Cubic design response surface approximation to the probability of failure and probabilistic sufficiency factor (calculated by Monte Carlo simulation of $1 \times 10^6$ samples)

<table>
<thead>
<tr>
<th></th>
<th>Probability of failure</th>
<th>Probabilistic sufficiency factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rsquare adj.</td>
<td>0.6452</td>
<td>0.9990</td>
</tr>
<tr>
<td>RMSE predictor</td>
<td>0.000211</td>
<td>0.00176</td>
</tr>
<tr>
<td>Mean of response</td>
<td>0.0000580</td>
<td>1.079</td>
</tr>
</tbody>
</table>

Table 9. Optimum panel design

<table>
<thead>
<tr>
<th>b</th>
<th>h</th>
<th>t₁</th>
<th>t₂</th>
<th>Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.70</td>
<td>1.9854</td>
<td>0.1005</td>
<td>0.1246</td>
<td>2581</td>
</tr>
</tbody>
</table>

Table 10. Probabilities of failure calculated by Monte Carlo simulation of $1 \times 10^6$ samples

<table>
<thead>
<tr>
<th>Probability of failure of load case 1</th>
<th>Probability of failure of load case 2</th>
<th>System probability of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>95×10^6</td>
<td>95×10^6</td>
</tr>
</tbody>
</table>